

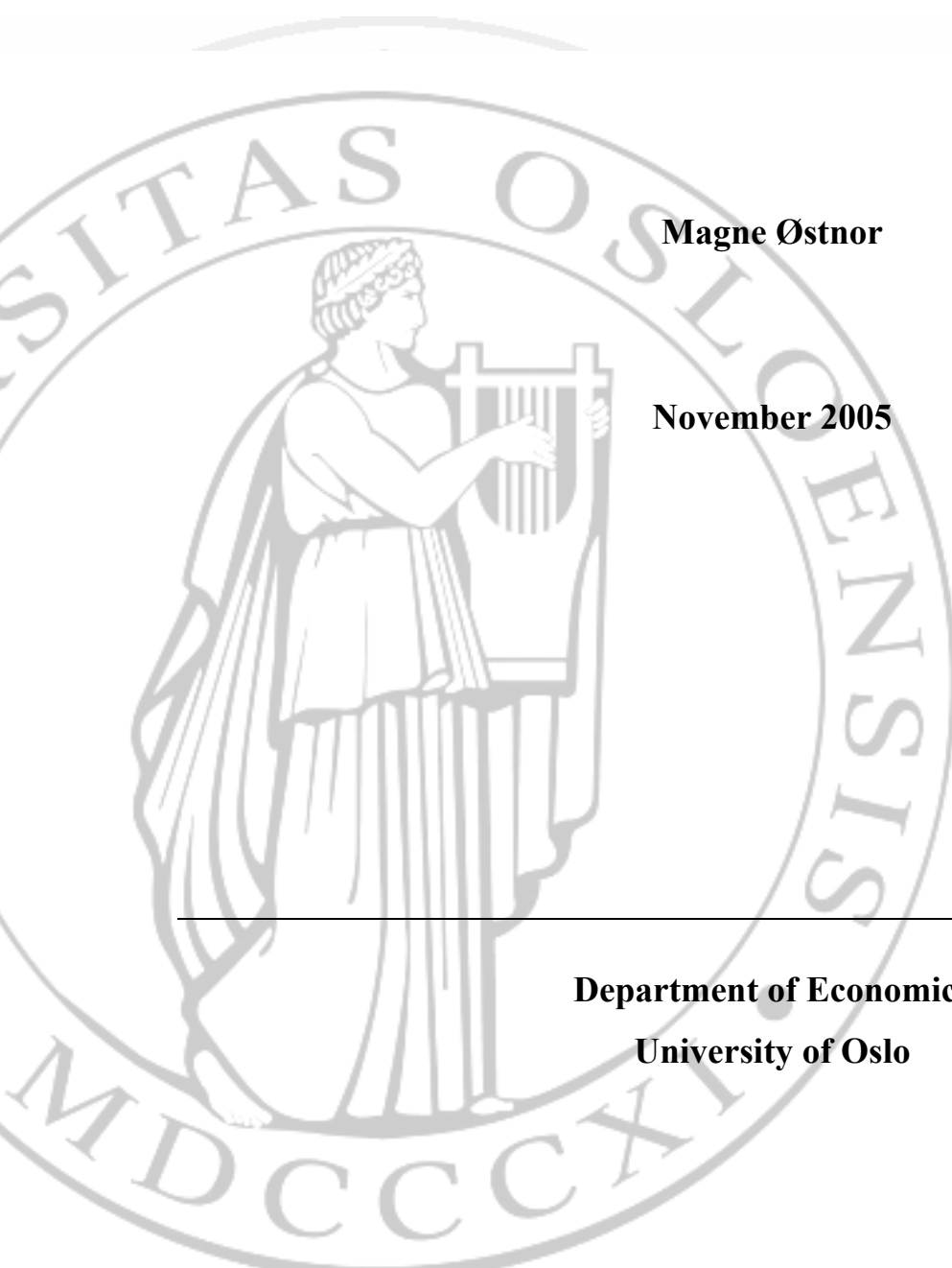
Master thesis for the Master of Philosophy in Economics degree

Monetary Policy, Models & short term Forecasting

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The views expressed in this thesis are those of the author, and should not be regarded as those of Norges Bank. Any remaining errors are of course the responsibility of the author.

Summary

Monetary policy in Norway is oriented towards low and stable inflation. Norges Bank operates a flexible inflation targeting regime so that variability in inflation and variability in output and employment are given weight in the interest rate reaction function. The interest rate is set with a aim to stabilize inflation at the target within a reasonable time horizon, normally 1-3 years. The relevant horizon will depend on the type of disturbances to which the economy is exposed to and how they affect the expected future path for inflation and the real economy.

Because monetary policy influences inflation and the real side of the economy with long and variable lags, an inflation-targeting central bank needs to be forward-looking and make projections of the economic development. Inflation targeting is by Svensson (1997) called inflation *forecast* targeting, because the intermediate target of monetary policy is the central banks' forecast of inflation. This calls for a good understanding of the current situation of the economy, in debt knowledge of the disturbances to which the economy is exposed to and a thorough understanding of the transmission mechanism of monetary policy. Thus, projections of the most relevant macroeconomic variables are of great importance as a foundation for deciding on the inflation forecasts, and in that, for the interest rate decisions made in order for inflation to reach its target.

The forecasts published by Norges Bank in the inflation reports are not the result of a single model, but rather forecasts based on subjective assessments between institutional knowledge and forecasts from both empirical and theoretical models (Qvigstad (2005)). The process of making projections can be viewed as a tripartite approach; the analysis of the current situation and the short-term outlook, the medium run and finally the long-run where one assumes steady-state and that certain economic relationships will hold. Central banks usually operate a variety of different models, which in different ways are useful for making projections at these horizons. A core model often constitutes the main part of the modelling framework providing the economic structure relevant for policy analysis. Such macroeconomic models are usually designed to make medium to long term projections, and support both policy and risk analysis, as well as the communication of monetary policy. The

analysis of the current situation and the short-term outlook, typically the next year, is normally based on all available relevant statistics, qualitative information and specialised forecasting tools, in addition to pure judgement. In the short run, models such as vector autoregression (VAR) models, autoregressive moving average (ARMA) models and also random walk models have proven to produce good forecasts of macroeconomic variables. In addition, they can help quantify the uncertainty related to different projections, and in so, help communicating the imprecision in policy making. Apart from being models for forecasting, these models can also contribute to a certain degree of consistency in the forecasting process, or as Adolfson et. al. (2005, p. 19) put it:

“Our results suggest, e.g., that subjective forecast often may be too myopic and not take enough account of important historical regularities in the data. (...) When the economists work with some common models they believe in, it is easier to avoid being trapped in inefficient “battles of anecdotes.”

Thus, forecasts from short term models can serve as cross-references and benchmarks of more judgemental forecasts and forecasts from other models, and also impose structure and consistency on the process of projection making.

To investigate the predictive accuracy of some commonly used short term forecasting models, we make forecasts of the monthly and twelve month domestic consumer price inflation and the unemployment rate using unrestricted VAR models, ARMA models and random walks. We use short run forecasting horizons of three, six and 12 months. In doing this we try to highlight the differences in predicting real and nominal variables by pure univariate models (ARMA models) and multivariate models (VAR models). While univariate models highlight only the statistical properties of a particular data series, multivariate models take into account more information and implicitly impose an economic relationship between the variables in the model. The forecasting accuracy of the different models is evaluated using the root mean square error, and the results are compared with forecasts from a pure random walk model. The analysis is applied to Norwegian monthly data.

We find the multivariate VAR models to be best at predicting the unemployment rate at all three horizons, outperforming the random walk, but also the ARMA models. As for the

inflation rates, ARMA outperforms the random walk at all horizons, while the VAR models perform similarly in predicting the twelve month inflation rate at the six and twelve month horizon. The reason for the forecasting performance of the inflation rate of the ARMA models can be attributed to the persistence in inflation.

A thorough understanding of the transmission mechanism of monetary policy is crucial in order for the central bank to be able to undertake the appropriate policy responses to shocks to the economy. Since Sims (1980), studies investigating the effects of monetary policy have to a large extent been undertaken using a structural VAR approach. While VAR studies of the closed economy have provided accepted empirical evidence of the effects of monetary policy shocks, similar studies of open economies have been less successful in providing a consensus regarding these effects. In fact, the literature has encountered several puzzles, much of which is a result of the introduction of the exchange rate. Bjørnland (2005a), on the other hand, finds much more theory consistent results, using Norwegian quarterly data. In particular, a contractionary monetary policy shock implies an immediate appreciation of the exchange rate, a temporary lowering of output, and a sluggish but negative effect on consumer price inflation.

By recursive identification of a VAR model, we analyze the effect of a shock to monetary policy using Norwegian monthly data. The recursive identification imposes a standard structure on the variables in the VAR, in that the macroeconomic variables react to shocks to monetary policy with a lag, while we allow for a contemporaneous effect of a shock to the macroeconomic variables on monetary policy. Our results show that a contractionary shock to monetary policy increases the domestic interest rate temporarily, while the exchange rate appreciates immediately, before it slowly depreciates back to baseline. Unemployment increases for a period of five quarters, while the effect on inflation is sluggish, but negative, with monetary policy having its full effect after two and a half years.

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1. Introduction

Monetary policy in Norway is oriented towards low and stable inflation. Norges Bank operates a flexible inflation targeting regime so that variability in inflation and variability in output and employment are given weight in the interest rate reaction function. The interest rate is set with a aim to stabilize inflation at the target within a reasonable time horizon, normally 1-3 years. The relevant horizon will depend on the type of disturbances to which the economy is exposed to and how they affect the expected future path for inflation and the real economy.

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The forecasts published by Norges Bank in the inflation reports are not the result of a single model, but rather forecasts based on subjective assessments between institutional knowledge and forecasts from both empirical and theoretical models (Qvigstad (2005)). The process of making projections can be viewed as a tripartite approach; the analysis of the current situation and the short-term outlook, the medium run and finally the long-run where one assumes steady-state and that certain economic relationships will hold. Central banks usually operate a variety of different models, which in different ways are useful for making projections at these horizons. A core model often constitutes the main part of the modelling framework providing the economic structure relevant for policy analysis. Such macroeconomic models are usually designed to make medium to long term projections, and support both policy and risk analysis, as well as the communication of monetary policy. The analysis of the current situation and the short-term outlook, typically the next year, is

normally based on all available relevant statistics, qualitative information and specialised forecasting tools, in addition to pure judgement. In the short run, models such as vector autoregression (VAR) models, autoregressive moving average (ARMA) models and also random walk models have proven to produce good forecasts of macroeconomic variables. In addition, they can help quantify the uncertainty related to different projections, and in so, help communicating the imprecision in policy making. Apart from being models for forecasting, these models can also contribute to a certain degree of consistency in the forecasting process, or as Adolfson et. al. (2005, p. 19) put it:

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Thus, forecasts from short term models can serve as cross-references and benchmarks of more judgemental forecasts and forecasts from other models, and also impose structure and consistency on the process of projection making.

To investigate the predictive accuracy of some commonly used short term forecasting models, we make forecasts of the domestic consumer price inflation and the unemployment rate using unrestricted VAR models, ARMA models and random walk. We use short run forecasting horizons of three, six and 12 months. In doing this we try to highlight the differences in predicting real and nominal variables by pure univariate models (ARMA models) and multivariate models (VAR models). While univariate models highlight only the statistical properties of a particular data series, multivariate models take into account more information and implicitly impose an economic relationship between the variables in the model. The forecasting accuracy of the different models is evaluated using the root mean square error, and the results are compared with forecasts from a pure random walk model, where the forecast for the next period is equal to this period's observation. The analysis is applied to Norwegian monthly data.

A thorough understanding of the transmission mechanism of monetary policy is crucial in order for the central bank to be able to undertake the appropriate policy responses to shocks to the economy. Since Sims (1980), studies investigating the effects of monetary policy have

to a large extent been undertaken using a structural VAR approach. While VAR-studies of the closed economy has provided accepted empirical evidence of the effects of monetary policy shocks, similar studies of open economies has been less successful in providing a consensus regarding these effects. In fact, the literature has encountered several puzzles, much of which is a result of the introduction of the exchange rate; see Eichenbaum and Evans (1995). Bjørnland (2005a) on the other hand, finds much more theory consistent results, using Norwegian quarterly data. In particular, a contractionary monetary policy shock implies an immediate appreciation of the exchange rate, a temporary lowering of output, and a sluggish but negative effect on consumer price inflation. Following the setup in Bjørnland (2005a), we identify the effects of a contractionary shock to the domestic interest rate using a structural VAR model. However, we use monthly Norwegian data from a single monetary policy regime; thereby explicitly examining whether the effects found in the literature are preserved in recent time.

The thesis is organised as follows. Section 2 reviews relevant literature on the effects of monetary policy shocks in both closed and open economies. The identification of the VAR model and the effects from the model of a contractionary monetary policy shock is thereafter analyzed. In section 3, the theory of forecasting with both multivariate and univariate models is presented, while sections 4 and 5 present the forecasting procedure and compare the results from the forecasting of the chosen variables. Section 6 concludes our findings¹. Appendix A offers a detailed description of the data used and the respective sources,² while Appendix B offers a description of the concept of stationarity, unit roots and cointegration.

¹ The empirical results are obtained using Census X-12-ARIMA 2.09, PcGive 10.1 and EViews 5.0.

² All data are seasonally adjusted by their respective sources. The dataset is available from the author upon request. Contact details: magne.ostnor@norges-bank.no

2. The structural VAR model

Studies investigating the transmission mechanism of monetary policy have to a large extent been undertaken using a vector autoregression (VAR) approach. The VAR approach initiated by Sims (1980) was introduced after the large structural macroeconomic models of the 1950's and 1960's had proven to be unsatisfactory. Sims argued that the restrictions used to identify these simultaneous equations models were “incredible”, because in a general equilibrium, all variables would affect all other variables. By imposing a recursive identification strategy between the macroeconomic and the monetary policy variables, Christiano, Eichenbaum and Evans (1996, 2005) have derived “stylized facts” on the effect of a contractionary shock to monetary policy in a closed economy. They conclude that models of the monetary transmission mechanism should after a contractionary policy shock be consistent with a temporary rise in the interest rates, an initially small but eventually negative response in inflation and a hump-shaped response in production, with no long-run effects of the shock.

While VAR studies of the closed economy have provided accepted empirical evidence of the effects of monetary policy shocks, similar studies of open economies have been less successful in providing a consensus regarding these effects, in particular with respect to the exchange rate. Traditional rational expectations overshooting models, such as the one considered by Dornbusch (1976), assume an impact appreciation, followed by a gradual and persistent depreciation of the domestic currency, as the positive interest rate differential would lead to expectations of a depreciation of the domestic currency in line with the uncovered interest rate parity. However, several studies (e.g. Eichenbaum and Evans (1995) and Lindé (2003)) find that if the exchange rate appreciates; it does so for a prolonged period of time, thereby essentially violating the uncovered interest rate parity condition. In the literature, this is referred to as the *forward discount rate puzzle*, or *delayed overshooting*.

However, although the VAR approach is frequently used in monetary policy analysis, it is also controversial. Critics claim that the identification needed to orthogonalize the structural shocks is just the kind of restrictions Sims deemed “incredible”. These structural VAR models are also criticized of being sensitive to misspecification, such as which variables that

are included and the specification of these variables. The VAR approach is also subject to the Lucas critique, i.e. that the coefficients describing the impact of a shock to the domestic interest rate on the other variables in the model depends on the monetary policy regime. Thus, a shift in policy regime could lead to parameter instability.

We base the choice of variables in the VAR model on the New-Keynesian type of small open economy models, in line with Clarida, Gali and Gertler (1999), Svensson (2000) and Bjørnland (2005a, 2005b). The VAR model comprises of the foreign interest rate (i^*), the registered unemployment rate (u), the twelfth difference of the log of the domestic consumer price index (π^y), the domestic interest rate (i) and the log of the real exchange rate (e) (cf. Appendix A for an exposition of the data and the respective sources).

2.1 Identification of the structural VAR

In order to be able to use VAR models for structural inference and policy analysis, that is to estimate the effect of exogenous shocks, the VAR model must be identified. Assuming stationarity of the y vector of N endogenous variables in the VAR model, the model can be written as

$$(2.1) \quad A(L)y_t = \alpha + u_t,$$

where α is a vector of constant terms and u_t is a vector of serially uncorrelated (white noise) residuals with covariance matrix Ω . The matrix $A(L)$ is a matrix polynomial in the lag operator, and can be written in the form

$$(2.2) \quad A(L) = \sum_{j=0}^p \mathbf{A}_j L^j = \mathbf{A}_0 - \mathbf{A}_1 L - \mathbf{A}_2 L^2 - \dots - \mathbf{A}_p L^p,$$

where \mathbf{A}_0 is the identity matrix, \mathbf{A}_j is the matrix of autoregressive coefficients at lag j . By the Wold decomposition theorem³ we can then rewrite the model in terms of its structural moving average, where $B(L) = A(L)^{-1}$

³ Any zero-mean covariance stationary process can be written as an infinite distributed lag of white noise

errors. That is: $X_t = \sum_{j=0}^{\infty} \psi_j \varepsilon_{t-j} + \kappa_t$, where $\psi_0 = 1$ and $\sum_{j=0}^{\infty} \psi_j^2 < \infty$.

$$(2.3) \quad \mathbf{y}_t = B(L)\mathbf{u}_t,$$

ignoring the constant term from here on. To obtain identification of the relevant structural parameters, given the estimation of the reduced form model, we have to impose plausible restrictions on the elements of the $B(L)$ matrix. This is done by assuming orthogonality of the structural disturbances and that these disturbances can be written as a linear combination of the innovations, that is

$$(2.4) \quad \mathbf{u}_t = \mathbf{S}\boldsymbol{\varepsilon}_t,$$

where $\boldsymbol{\varepsilon}_t$ is the vector of uncorrelated disturbances, which are normalized to have unit variance. The VAR model can then be expressed as

$$(2.5) \quad \mathbf{y}_t = B(L)\mathbf{S}\boldsymbol{\varepsilon}_t,$$

where we impose the identifying restrictions on \mathbf{S} to isolate the structural parameters. From the normalisation of $\boldsymbol{\varepsilon}_t$ we have that $\mathbf{S}\mathbf{S}' = \boldsymbol{\Omega}$. This normalisation imposes $N(N+1)/2$ restrictions on the matrix \mathbf{S} of the N^2 restrictions needed to identify the matrix. Hence, additional restrictions are needed to fully identify \mathbf{S} . By the recursive Cholesky decomposition⁴, we assume that macroeconomic variables cannot react simultaneously to policy shocks, while policy variables are allowed to react contemporaneously to changes in the macroeconomic variables, which is an often used assumption in the VAR literature (e.g. Sims (1980) or an open economy application such as Lindé (2003)). Hence, the identification of the structural shocks depends on the ordering of the variables. This then places the remaining $N(N-1)/2$ restrictions needed to identify the \mathbf{S} matrix.

The use of recursive identification will as shown by Bjørnland (2005a, 2005b) put a zero restriction on the contemporaneous effect of the exchange rate on the interest rate, assuming the causal ordering of the domestic interest rate and the exchange rate. By imposing a long-run restriction on the exchange rate, and in doing so, allowing the domestic interest rate to

⁴ This just-identification scheme of structural shocks is based on the Cholesky-decomposition of matrices. The most endogenous variables are ordered last, thereby imposing a recursive economic structure.

react simultaneously to news in the exchange rate, Bjørnland finds the interdependence between the interest rate and the exchange rate to increase considerably. Using quarterly data, it would be unreasonable to assume that policymakers do not react to surprise exchange rate movements occurring in the period in which the interest rate decisions are made. Allowing for contemporaneous correlation between the exchange rate and the interest rate is therefore reasonable. However, this might not be a profound problem with higher frequency data. As the Executive Board of Norges Bank meets every sixth week, we would expect the board to emphasize and react to the latest trends in the exchange rate markets, and not the very latest developments. Bagliano and Favero (1998, p. 1073) find the standard Cholesky recursive decomposition using monthly data “*consistent with a wide spectrum of alternative theoretical structures, and imply a minimal assumption on the lag of the impact of monetary policy actions on macroeconomic variables*”. We therefore use the just-identification scheme of the Cholesky decomposition, which puts the following restrictions on the \mathbf{S} matrix:

$$(2.6) \quad \begin{bmatrix} i^* \\ u \\ \Pi_D \\ i \\ e \end{bmatrix} = B(L) \begin{bmatrix} S_{11} & 0 & 0 & 0 & 0 \\ S_{21} & S_{22} & 0 & 0 & 0 \\ S_{31} & S_{32} & S_{33} & 0 & 0 \\ S_{41} & S_{42} & S_{43} & S_{44} & 0 \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} \end{bmatrix} \begin{bmatrix} \varepsilon^{i^*} \\ \varepsilon^u \\ \varepsilon^{\pi_D} \\ \varepsilon^i \\ \varepsilon^e \end{bmatrix}$$

The foreign interest rate is placed first, which is a common assumption in the analysis of a small open economy. The ordering of the unemployment rate and the inflation rate is somewhat arbitrary, but altering the ordering does not make a lot of difference. By the above ordering of the \mathbf{S} matrix, we have the vector ε_t of orthogonal disturbances as $\varepsilon_t = [\varepsilon_t^{i^*}, \varepsilon_t^u, \varepsilon_t^{\pi_D}, \varepsilon_t^i, \varepsilon_t^{ER}]$, where the shock of interest is the monetary policy shock ε_t^i .

Once \mathbf{S} is identified, there are several ways of examining the effect of exogenous shocks to the endogenous variables. The time series of the exogenous shocks can be obtained, or the forecast error variance decomposition can be used to determine the proportion of the forecast error variance decomposition of each endogenous variable attributable to each shock at different forecast horizons. We will however concentrate on impulse responses, which are used to trace out the dynamic effect of a shock to the interest rate on the other endogenous variables over time.

2.2 Effects of a monetary policy shock

A VAR model investigating the effect of a shock to monetary policy is usually estimated over a period with a single monetary policy regime. In the case of Norway, an inflation targeting regime officially replaced a managed float regime in March 2001. Prior to that, Norges Bank aimed at stabilizing the krone against European currencies. However, in January 1999, the central bank announced that the best way to stabilize the exchange rate against European currencies was by keeping inflation low at the level of the Euro-countries (Gjedrem (1999)). Prior to 1999, the interest rate was in periods of depreciation pressure increased, thus the interest rate and the exchange rate moved in the same direction in these periods of depreciation pressure. The VAR model is therefore estimated using a monthly data sample from 1999m1 to 2005m7 which coincides with the period in which Norges Bank has aimed at stabilizing inflation.

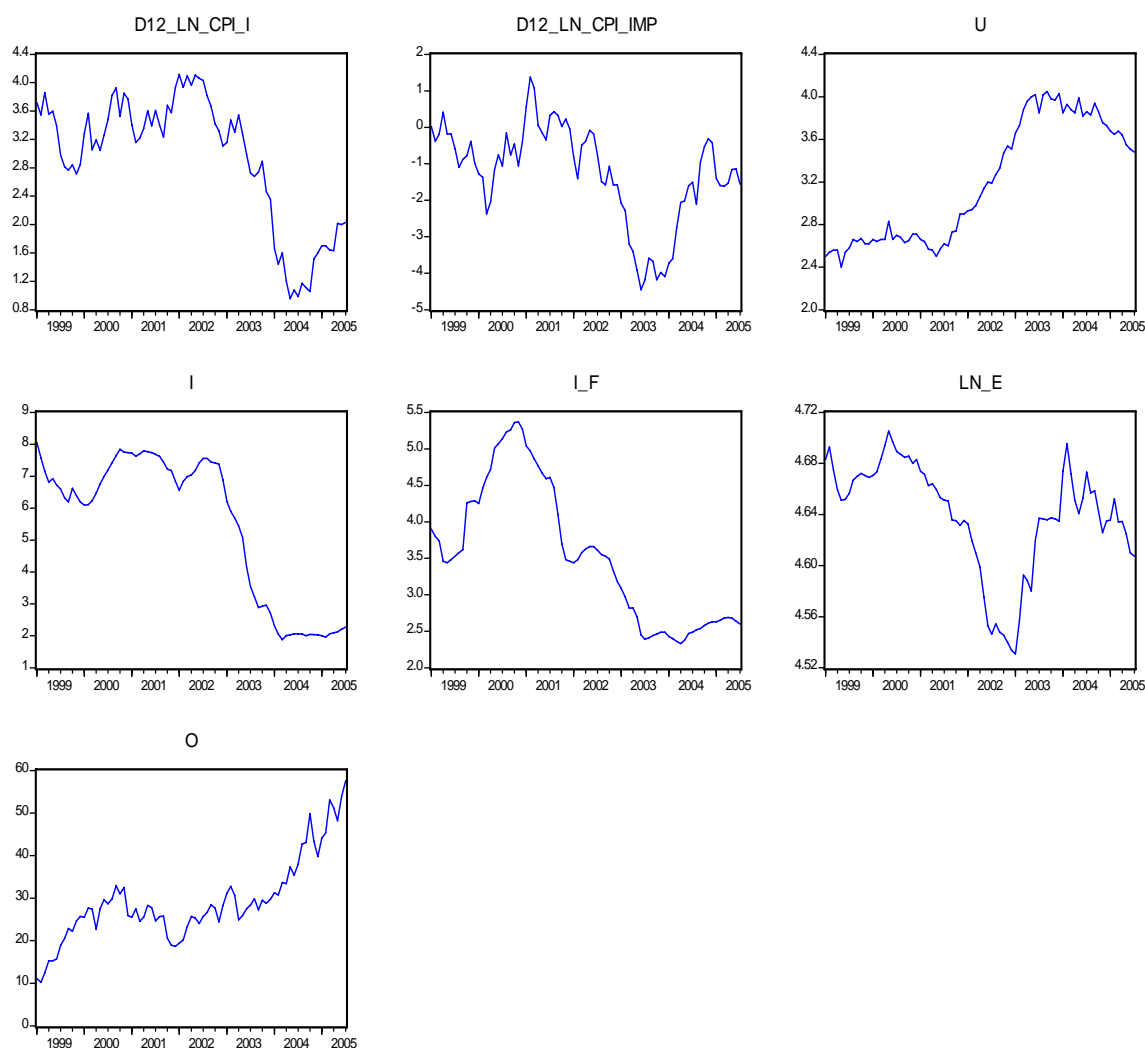


Figure 2.1 Time series plots of all variables, 1999m1-2005m7.

We make the usual log transformation of the trending series (P_d , P_{imp}) and the exchange rate, leaving the unemployment rate and the domestic and foreign interest rates in levels. The domestic and imported consumer price indexes are represented in twelfth differences.

From Figure 2.1, we see that some variables display possible nonstationary behaviour. Testing for unit roots using the augmented Dickey-Fuller test, all variables are found to be $I(1)$ in the sample period, cf. Table 2.1. There is little evidence of cointegration, with tests for cointegration accepting the hypothesis of at most one cointegration vector, cf. Table 2.2⁵. However, we have to bear in mind the low power of unit root and cointegration tests using such a short sample. Furthermore, seasonally adjusted data will, as shown by Ghysels, Lee and Noh (1994), reduce the power of the unit root tests further. Hence, we chose to model the data in levels. Any cointegrating relationship will then implicitly be determined in the model. This is consistent with other relevant studies of the transmission mechanism of monetary policy.

Table 2.1 Augmented Dickey-Fuller test, 1999m1-2005m7^a

	π_d^y	π_{imp}^y	u	i	i*	e	o
	t-adj	t-adj	t-adj	t-adj	t-adj	t-adj	t-adj
Levels	-1,13 (0)	-1,77 (0)	-1,22 (0)	-1,09 (1)	-0,71 (1)	-1,92 (1)	-1,43 (0)
1 st difference	-8,92* (0)	-7,88* (0)	-10,24* (0)	-4,70* (0)	-6,17* (0)	-7,20* (0)	-9,62* (0)

^a Except for the level of the oil price and the exchange rate which are tested for nonstationarity with a constant and a trend, the series are tested with a constant only. Critical values are taken from MacKinnon (1996). The number of lags in the ADF test are reported in parenthesis.

* Significant at the 1% level.

Table 2.2 Johansen cointegration test, 1999m1-2005m7

	Trace test	p-value	Maximum eigenvalue	p-value
None cointegration vector	124,58	0,00	77,10	0,00
At most one cointegration vector	47,48	0,05	26,88	0,06

When determining the lag order in the VAR, we test for lag reduction using Akaike (AIC), Schwarz (SIC) and Hannan-Quinn (HQ) information criteria. The intention of these criteria

⁵ Cf. appendix B for an exposition of the concept of stationarity, unit roots and cointegration.

is to provide the user with a statistic that strikes a balance between the specification of the model and the goodness of fit. They are widely used when analyzing time series to choose the number of distributed lags.

(i) Akaike Information Criterion (AIC)

$$(2.7) \quad AIC = \ln\left(\frac{e'e}{n}\right) + \frac{2K}{n}$$

(ii) Schwarz Information Criterion (SIC)

$$(2.8) \quad SIC = \ln\left(\frac{e'e}{n}\right) + \frac{K * \ln(n)}{n}$$

(iii) Hannan-Quinn Information Criterion (HQ)

$$(2.9) \quad HQ = \ln\left(\frac{e'e}{n}\right) + \frac{2K * \ln(\ln(n))}{n},$$

where $e'e$ is the sample sum of squares, K is the number of regressors and n is sample size. There are different formulations of these criteria, but the above are the ones reported in the software used. The number of lags to include in the model is chosen by minimizing the criterion over different number of lags. The three criteria differ only in the way they adjust for the number of parameters which are estimated. Hannan (1980) shows that if the true model is contained in the set of models examined, all these criteria will, as $n \rightarrow \infty$, lead to an overparameterised model. However, both the SIC and the HQ are strongly consistent in that they determine the true model asymptotically, and will therefore be the criteria of focus. Note that the criteria depend on the unit of measurement, and can therefore not be used to make choices between models with different transformations of the variables.

While the SIC and the HQ criteria suggested a lag reduction to only one lag, the AIC suggested that a reduction to twelve lags was acceptable at the one percent level. However, the residuals of the model at lower lags are correlated. To avoid the residual correlation, and

to reduce the chance of overfitting, we chose five lags which is the lowest lag order model where the hypothesis of no autocorrelation is not rejected.⁶

When specified in levels with five lags, no roots of the VAR lies outside the unit circle, i.e. the VAR satisfies the stability condition, and invertibility is thus ensured. This indicates that even though all variables are found to be I(1) and the Johansen test for cointegration indicates only one cointegration vector, there exists a cointegrating relationship between the variables which ensures stationarity of the model. Thus, we could have estimated the model imposing this relationship. However, estimating this relationship is difficult, and imposing the wrong relationships would give random and spurious results.

Box 1. Testing for residual misspecification.

Autocorrelation

The Ljung-Box Q-statistic (cf. Ljung and Box (1978)) is a test often used to test whether a series is white noise. It is a refinement of the Box-Pierce statistic (cf. Box and Pierce (1970)), and is supposed to have better finite sample properties. At k lags, it tests the null hypothesis of no autocorrelation up to lag k, and is computed as:

$$(2.10) \quad Q_{LB} = N(N+2) \sum_{j=1}^k \frac{\tau_j^2}{N-j} \quad Q_{LB} \sim \text{asym}(\chi_k^2),$$

where N is the number of observations and τ_j is the j-th autocorrelation. The number of lags, k, is determined by the specification of the model. If the series tested are residuals from ARIMA estimation, the degrees of freedom are adjusted by the number of AR and MA components.

Despite its wide acceptance in applied time series econometrics, critics claim the Q-statistic inappropriate in autoregressive models, because lagged dependent variables biases the residual autocorrelation towards zero. Maddala (2001) discuss some limitations of this statistic, and suggests using a LM test to test for high-order autocorrelation. However, because of its widespread use and out of computational ease, the Q-statistic

⁶ This is not very different from other VAR studies of monetary policy using monthly data. Eichenbaum and Evans (1995), Baglioni and Favero (1998) and Kim and Roubini (2000) use six lags in their analysis.

will be our choice of method when testing for residual autocorrelation in the random walk and ARMA-models, but we will use the LM test when testing for autocorrelation in the VAR models.

Normality

Testing for normality is done by testing if the skewness and kurtosis of the residuals (the third and fourth order moment) corresponds to those of the normal distribution. While Jarque-Bera is a commonly used test statistic, Shenton and Bowman (1977) question its small sample properties and suggest a statistic which is asymptotically equal to the J-B, but is shown to be closer to standard normal in small samples. The statistic is asymptotically $\chi^2(2k)$. The multivariate equivalent to the test of normality, transform the residuals into independent normal residuals, before calculating the univariate skewness and kurtosis. The multivariate statistic is $\chi^2(2k)$, where k is the number of equations in the system, and is invariant to the ordering of the equations in the system. For a thorough understanding of the test, cf. Doornik and Hansen (1994).

Heteroskedasticity

The test for heteroskedasticity is based on White (1980) and is performed by regressing the squared residuals on the original regressors and the squares of these regressors. Regressors that are redundant when squared or because of multicollinearity, are left out. The null hypothesis is unconditional homoskedasticity, and assuming the regression has n regressors plus a constant term; the test statistic is distributed as $\chi^2(mn)$, where n is the number of regressors, and $m = \frac{k(k+1)}{2}$. When testing for heteroskedasticity in a multivariate framework, the test regresses the error variances and covariances on the original regressors and their squares. It is worth noting that small sample sizes and large number of lags can make calculation of the test impossible, because of the large number of parameters to be estimated.

Tests of residual misspecification reject the hypothesis of no autocorrelation and no heteroskedasticity (cf. Box 1 for an exposition of tests for residual misspecification, and Table 2.3 for results). However, the residuals are not normal, which is mainly due to excess

kurtosis in the interest rates equations. Tests of parameter stability suggest that the interest rate equations are stable, hence the remaining non-normality is ignored.⁷

Table 2.3 Tests of residual misspecification

	Autocorrelation	Heteroscedasticity	Normality	Skewness	Kurtosis
	LM(25)	$\chi^2(750,79)$	$\chi^2(10,79)$	$\chi^2(5,79)$	$\chi^2(5,79)$
Test statistic	23,19	774,89	46,76	5,15	41,62
p-value	0,5664	0,2569	0,00	0,40	0,00

The impulse responses of a one standard deviation shock to the domestic interest rate on all variables in the VAR are given in Figure 2.2. The figure also reports the asymptotic two standard-error-bands. The contractionary shock to monetary policy increases the domestic interest rate temporarily for a period of nearly half a year, after which the standard deviation bands include zero. The exchange rate initially appreciates, reaching its maximum after 2-3 months, and then slowly depreciates back to baseline. The unemployment rate increases, reaching its maximum after almost five quarters, before the effect of the interest rate shock dies out. Domestic inflation initially increases, before it falls, with monetary policy having its full effect on inflation after two and a half years. The initial rise in prices following a contractionary monetary policy shock has been referred to as a price puzzle in the literature, cf. Eichenbaum (1992). However, the rise is not significant, and is therefore not any evidence of such a puzzle.

Compared to the results of Bjørnland (2005a), which is another study applied to Norwegian data, these results are not very different. Using a different structural identification scheme, allowing for simultaneous interaction between the domestic interest rate and the exchange rate, Bjørnland finds the same temporary increase in the domestic interest rate over a period of a year. After a 1 percentage point increase in the interest rate, the real exchange rate appreciates by approximately 0.8 percent after 1 quarter. The quantitative effect on the exchange rate of a contractionary shock to monetary policy in our model is somewhat stronger, and the adjustment back to baseline is slower. However, this can be attributed to

⁷ We perform rolling tests of parameter stability, testing the residual variance within the estimation period and forecast period, using a window of 24 observations. For a thorough exposition of the 1-step Chow test, the breakpoint Chow test, and the N-step Chow test, cf. Doornik and Hendry (2001b)

Response to Cholesky One S.D. Innovations ± 2 S.E.

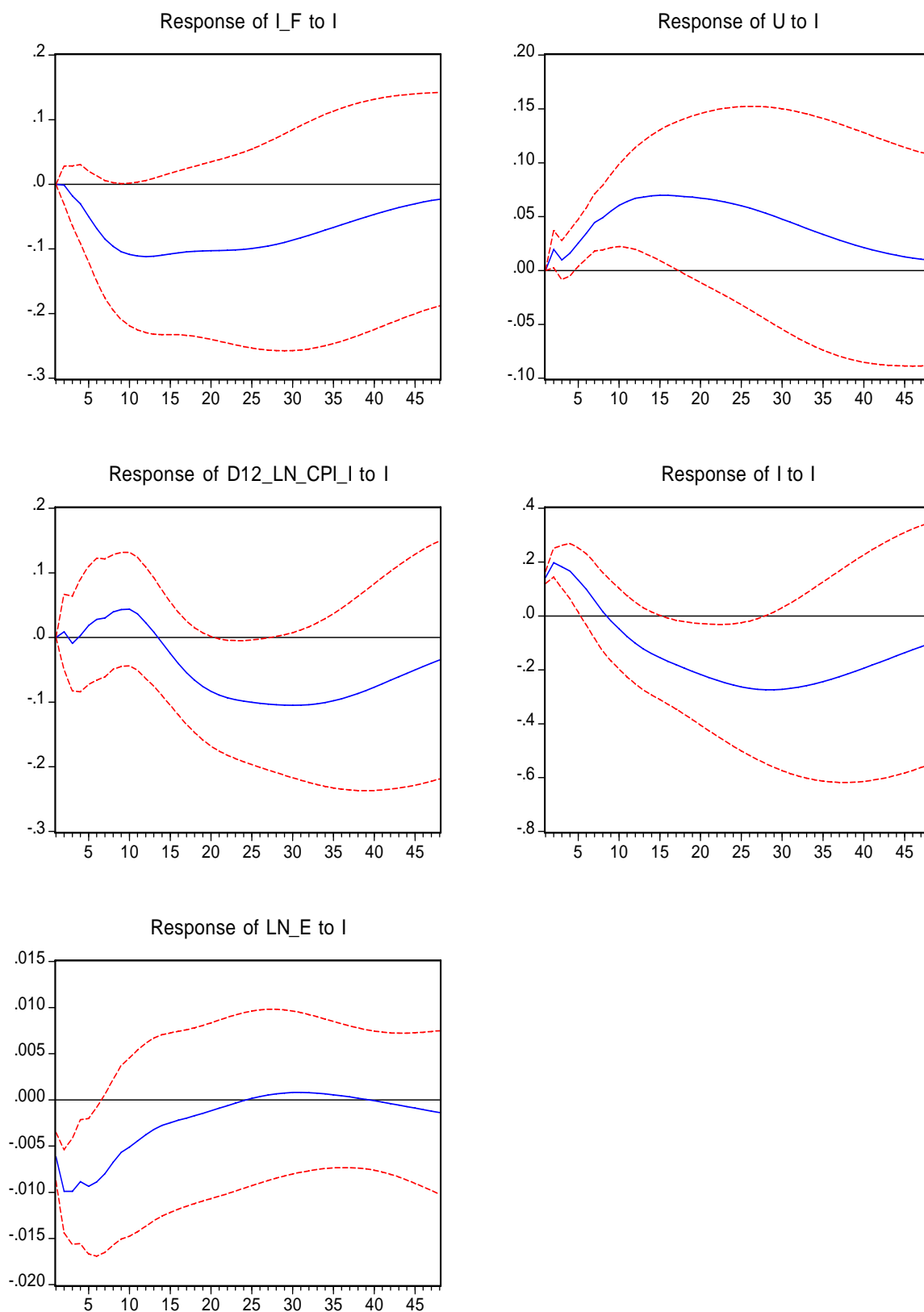


Figure 2.2 Responses to a monetary policy shock, 1999m1-2005m7.

the higher degree of noise in monthly data, and that the use of quarterly data not will capture the full effect in the exchange rate. The maximum effect on inflation is reached after nearly three years, while the maximum effect on GDP is reached after 5 quarters, which is broadly in line with our findings. Hence, it seems that using monthly data on a small sample reflecting a single monetary policy regime, the Cholesky decomposition seems to capture the feature of a monetary policy shock that was found in the structural VAR using quarterly data in Bjørnland (2005a), with a somewhat earlier effect on inflation.

2.3 Robustness of results

Following Olsen, Qvigstad and Røisland (2003) who find that with the possible exception of the brief period from late 1996 to late 1998, monetary policy in Norway has followed some kind of Taylor rule from 1993; we could have estimated the model with a larger data sample. By estimating the model with data from 1994⁸, and include a dummy taking the value of 1 in the period from 1996m10 through to 1998m12, and a dummy in the period of 2002m4-2002m6 which corresponds to a severe appreciation of the krone, the results are qualitatively the same. However, the exchange rate initially depreciates after a contractionary monetary policy shock, and remains depreciated for three months before appreciating for a period of nine months. These effects are however quantitatively smaller than with the shorter sample, and also not significant. The choice of the longer sample period will moreover make the model subject to the Lucas critique.

We also check for robustness of the above results with respect to additional variables. Figure (2.3) graphs the effect of a one standard deviation shock to the interest rate on the exchange rate in the baseline model, as well as when the oil price and the imported inflation rate are included. The two standard error bands are of the baseline model. All in all, the effects on the exchange rate remain as in the baseline VAR. When we include the oil price, the real exchange rate depreciates above the baseline after two years. However, this is within the two standard error bands of the baseline model. Thus, the results remain qualitatively similar when additional variables are included.

⁸ Using the twelve month inflation rates, estimating the model with data from 1994m1 will include data from 1993.

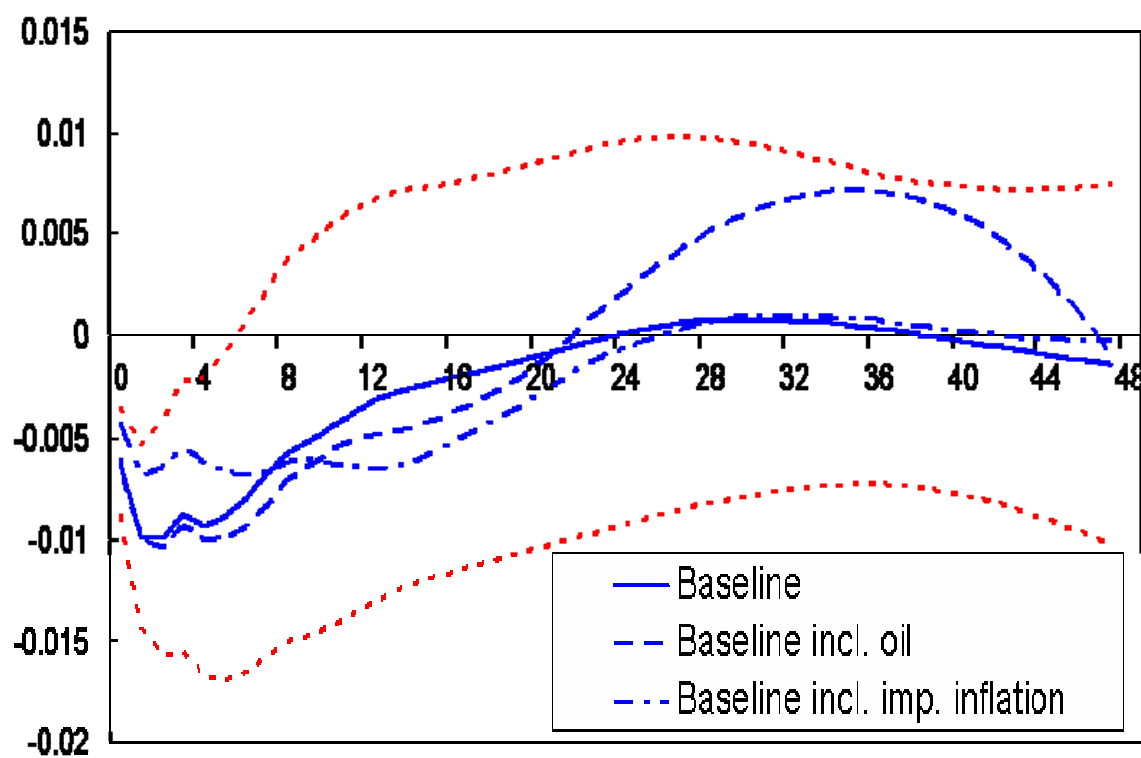


Figure 2.3 Response on the exchange rate of a contractionary monetary policy shock, using different variables in the model. 1999m1-2005m7.

3. Forecasting theory and methodology.

From the results in the structural VAR model, we see that after a contractionary shock to monetary policy, the response in the inflation rate is delayed compared to the response in the unemployment rate. This persistence in the inflation rates may be attributed to the slow adjustment of price-setters to shocks, also known as nominal rigidities. Due to this persistence in prices, we might expect that a univariate model might perform well in forecasting inflation because these models highlight only the historical variation in the variable. The real side of the economy however, reacts fairly instantly to a policy shock, and including more information when forecasting e.g. unemployment may therefore improve forecasts, even in the short run.

3.1 Forecasting using vector autoregressive models

Vector autoregressive models are a multiple time series generalisation of the univariate AR model. All variables in the system are endogenous, with each endogenous variable being a linear function of its own lagged values and lagged values of all the other variables in the system. Formally, a vector autoregression can be written as

$$(3.1) \quad \mathbf{y}_t = \boldsymbol{\alpha} + \sum_{s=1}^L \boldsymbol{\pi}_s \mathbf{y}_{t-s} + \mathbf{u}_t,$$

where \mathbf{y}_t is the $N \times 1$ vector of endogenous variables, $\boldsymbol{\pi}_s$ is a $N \times N$ matrix of coefficients, \mathbf{u}_t is a vector of serially uncorrelated (white noise) residuals with covariance matrix $\boldsymbol{\Omega}$ and s is the number of lagged dependent variables in the model. Using the lag operator, the reduced form VAR can be expressed as

$$(3.2) \quad A(L)\mathbf{y}_t = \boldsymbol{\alpha} + \mathbf{u}_t$$

Assuming the vector \mathbf{y}_t to be a vector of stationary variables, the VAR model will also be stationary⁹. The one-step-ahead forecast of the vector of endogenous variables is simply

⁹ Cf. Appendix B for an exposition of stationarity.

$$(3.3) \quad \mathbf{f}_{t,1} = \hat{\mathbf{a}} + \sum_{s=0}^L \hat{\boldsymbol{\pi}}_s \mathbf{y}_{t-s}$$

The h-step-ahead static forecast is

$$(3.4) \quad \mathbf{f}_{t,h} = \hat{\mathbf{a}} + \sum_{j=t+1}^{t+h} \hat{\boldsymbol{\pi}}_j \mathbf{f}_{t,j} + \sum_{s=0}^L \hat{\boldsymbol{\pi}}_s \mathbf{y}_{t-s}$$

Static forecasts imply that, when $h > 0$, the parameter estimates are not updated over the forecast period. Suppose our observed series at time $t+h$ are realisations from a general VAR model. The forecast error can be shown to be

$$(3.5) \quad \mathbf{e}_{t,h} = \mathbf{y}_{t,h} - \mathbf{f}_{t,h}.$$

For a thorough understanding of forecasting uncertainty, cf. Clements and Hendry (1998, Ch. 7.)

Critiques of the VAR approach claims that forecasts from the models are not robust to changes in the number of variables or the number of lags. The choice of variables in this paper is done to reflect a New-Keynesian small open economy model, like the ones presented in Clarida et al. (1999), Svensson (2000) and Bjørnland (2005a, 2005b). For forecasting purpose we will estimate different VAR models, using different variables, choosing the model that minimizes the predictive failure. Section 4.1 gives a more detailed exposition of the different models estimated and the choice of lag length. If too many lags or variables are included in the VAR model, better known as overfitting, forecasts from the model might perform poorly even if the model fit the data well through the estimation period. This will be the case if the model picks up systematic relationships as well as noise in the data. Thus, specifying a VAR will be striking a balance between the chance of overfitting and the need for keeping some dynamics. This has led to the development of Bayesian VAR (BVAR) models, where the coefficients in the VAR model are given some prior distributions. Litterman (1986) is a good exposition of forecasting using BVAR models. Estimating BVAR models would have been a useful comparison to the derived

impulse response functions, as well as to the forecasts made, but is considered beyond the scope of this thesis.

3.2 Forecasting using autoregressive moving average models

Unlike other methods of forecasting, ARMA models do not assume knowledge of any underlying economic model or structural relationship. It is assumed that past values of the series and past errors contain information about the future path of the series. Thus, forecasting from ARMA models are in essence a sophisticated form of extrapolation. ARMA models have been shown to outperform more sophisticated structural models in terms of short term forecasting ability, see for example Stockton and Glassman (1987) and Littermann (1986).

An autoregressive moving average model ARMA(p,q) is defined as:

$$(3.6) \quad X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \dots + \alpha_p X_{t-p} + \varepsilon_t + \beta_1 \varepsilon_{t-1} + \beta_2 \varepsilon_{t-2} + \dots + \beta_q \varepsilon_{t-q},$$

where $\{\varepsilon_t\}$ is a white noise process. Using the lag operator, (3.6) can be written as:

$$(3.7) \quad \phi(L)X_t = \theta(L)\varepsilon_t,$$

where $\Phi(L)$ and $\theta(L)$ are the polynomials of order p and q, respectively, defined as:

$$(3.8) \quad \phi(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p$$

$$(3.9) \quad \theta(L) = 1 + \beta_1 L + \beta_2 L^2 + \dots + \beta_q L^q$$

For stationarity we require the roots of $\phi(L) = 0$ to lie outside the unit circle. Invertibility of the MA component requires the roots of $\theta(L) = 0$ to lie outside the unit circle.¹⁰ Assuming invertibility of the MA component of the model, combinations of relatively small values of the lag parameters, p and q, represent a surprisingly wide variety of time series structures.

¹⁰ If the MA component is invertible, it can be rewritten as an infinite AR process.

This is usually explained by applying the Wold representation, that any zero-mean, weakly stationary process can be written as a linear combination of uncorrelated random variables.¹¹

We assume that the time series, if nonstationary, can be made stationary by differencing, i.e. that they are difference stationary. The ARIMA(p,d,q) model can thus be expressed as

$$(3.10) \quad \phi(L)[(1 - L^d)X_t] = \theta(L)\varepsilon_t,$$

where d is the order of integration needed to make the time series X_t stationary. If the series exhibit seasonal fluctuations, the seasonal pattern can be modelled in the same way. Thus the general ARIMA(p,d,q)(P,D,Q)₁₂ model can be expressed as:

$$(3.11) \quad \phi(L)\Phi(L^{12})[(1 - L^d)(1 - L^{12})X_t] = \theta(L)\Theta(L^{12})\varepsilon_t,$$

where $\Phi(L^{12})$ and $\Theta(L^{12})$ are the seasonal AR and MA components, and $(1 - L^{12})$ is the seasonal differences. However, the series used in this thesis are all seasonally adjusted, and the analysis will therefore be restricted to the class of models in (3.7).

The approach originally suggested by Box and Jenkins (1976) to select the appropriate ARMA model to fit the stationary series, implies a thorough investigation of the plots of the sample autocorrelation and partial autocorrelation functions, and inferring from patterns observed in these functions the correct number of lag parameters. While manageable when one has a pure AR or MA process, this approach becomes difficult and highly subjective when dealing with combined ARMA models. Model identification becomes an iterative process, where a more formal assessment of each model's residuals is the basis for model selection. However, to avoid the judgemental procedure of the Box-Jenkins approach, and out of computational ease, model selection is done on basis of predictive accuracy.

An alternative way of choosing p and q in the ARMA models or the number of lags in the VAR models would have been to use prior information from real business cycle (RBC) or

¹¹ In principle, finding the Wold representation implies fitting an infinite number of parameters to the data. However, in applications one needs to make some additional assumptions on the nature of the parameters. For a more detailed exposition, see for example Hamilton (1994). We make use of the Wold representation to limit the number of lag parameters, p and q, in the models tested.

dynamic stochastic general equilibrium (DSGE) models to determine the appropriate number of lags. The same models may also be used to estimate the corresponding coefficients. Lees and Matheson (2005) have found support of improved ARMA forecasts of post-war US GDP, using information from a RBC model. For other examples see Lees and Matheson (2005) and the references therein.

Suppose that our observed series is considered a realisation from the general ARMA (p,q) process. The linear representation of the future value x_{t+h} is then

$$(3.12) \quad x_{t+h} = a_{t+h} + \psi_1 a_{t+h-1} + \dots + \psi_{h-1} a_{t+1} + \psi_h a_t + \psi_{h+1} a_{t-1} + \dots,$$

where $\psi(L) = \phi^{-1}(L)\theta(L)$. Our forecast of x_{t+h} ($h \geq 1$) is made at time n , so only x_t, x_{t-1}, \dots is known. The forecast of x_{t+h} will be a linear combination of the past and present values of x , so that the forecast can in fact be regarded as the conditional expectation of x_{t+h} given x_t, x_{t-1}, \dots . Our h -step forecast at time t , can then be represented as

$$(3.13) \quad \begin{aligned} f_{t,h} &= E[x_{t+h} | x_t, x_{t-1}, \dots] \\ &= E[(a_{t+h} + \psi_1 a_{t+h-1} + \dots + \psi_{h-1} a_{t+1} + \psi_h a_t + \psi_{h+1} a_{t-1} + \dots) | x_t, x_{t-1}, \dots] \end{aligned}$$

Further, we know that

$$(3.14) \quad E[a_{t+j} | x_t, x_{t-1}, \dots] = \begin{cases} a_{t+j}, & j \leq 0 \\ 0 & j > 0 \end{cases},$$

since past values of a_{t+j} are known, and future values even though they are unknown, have zero expectation. Hence the forecast

$$(3.15) \quad f_{t,h} = \psi_h a_t + \psi_{h+1} a_{t-1} + \dots$$

can be shown to be the minimum mean square error forecast of x_{t+h} at time t . The forecast error h -step ahead is given by

$$(3.16) \quad e_{t,h} = x_{t,h} - f_{t,h} = a_{t+h} + \psi_1 a_{t+h-1} + \dots + \psi_{h-1} a_{t+1}$$

Taking expectations, we see that the forecast $f_{t,h}$ is an unbiased forecast. The variance of the forecast error is then

$$(3.17) \quad \text{var}(e_{t,h}) = \sigma^2(1 + \psi_1^2 + \psi_2^2 + \dots + \psi_{h-1}^2),$$

which is a linear combination of the unobservable future shocks after time t . However, while the one step ahead forecast errors ($e_{n,1} = a_{n+1}$) are uncorrelated, h -step ahead forecasts made at different time, or the forecasts for different horizons made at the same time, are not. The two correlations coefficients are given by

$$(3.18) \quad \rho(e_{t,h}, e_{t-j,h}) = \begin{cases} \frac{\sum_{i=j}^{h-1} \psi_i \psi_{i-j}}{\sum_{i=0}^{h-1} \psi_i^2} & , 0 \leq j < h \\ 0 & \text{otherwise} \end{cases}$$

$$(3.19) \quad \rho(e_{t,h}, e_{t,h+j}) = \frac{\sum_{i=0}^{h-1} \psi_i \psi_{i+h}}{\left(\sum_{i=0}^{h-1} \psi_i^2 \sum_{m=0}^{h+j-1} \psi_m^2 \right)^{1/2}}$$

As a result of this, the forecast function will have a tendency to lie either above or under the future values of x when they become observable.

As should be no surprise, the ARMA models are not very good at forecasting turning points. Because these models are estimated using only historical variation of the time series, structural shifts will only be predictable if they constitute a trend reversion. The models of low lag order will therefore usually not capture the business cycles, or any change in these cycles. However an argument for models of higher lags, these models are models intended for short term analysis, and are hence not expected to capture long term trends. The economic interaction between the variables in a VAR model will probably make such models better at predicting turning points. An extension to our analysis of predictive

accuracy would therefore be to evaluate the models on how well they predict these turning points, not solely relying on the conventional measures of evaluating predictive accuracy.

3.3 Forecasting using a random walk

A time series $\{X_t\}$ is said to be a pure random walk if

$$(3.20) \quad X_t = X_{t-1} + \varepsilon_t,$$

where $\{\varepsilon_t\}$ is white noise. Thus, the forecast of period $t+h$ made at period t is simply

$$(3.21) \quad \hat{X}_{t+h} = X_t$$

If we allow for a constant in the time series $\{X_t\}$, the random walk with drift can be expressed as

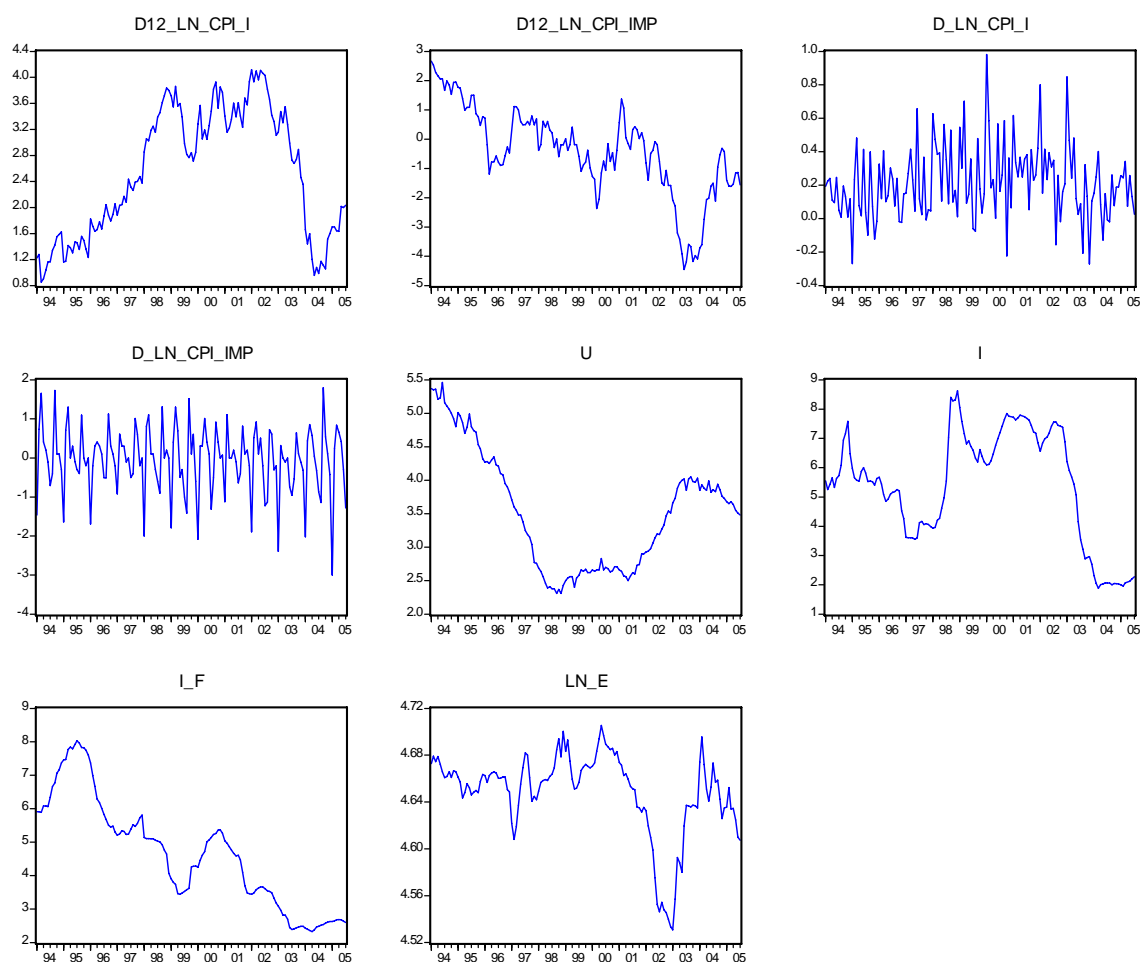
$$(3.22) \quad X_t = \mu + X_{t-1} + \varepsilon_t,$$

where $\{\varepsilon_t\}$ is white noise, and the corresponding h period forecast is thus

$$(3.23) \quad \hat{X}_{t+h} = n\mu + X_t$$

4. Forecasting in the short-medium run

We make forecasts of the unemployment rate and the monthly and twelve month domestic inflation rates at the three, six and 12-months horizons, using a recursive estimation scheme. To allow for more observations when making forecasts using autoregressive models, and following the findings in Olsen, Qvigstad and Røisland (2003), that monetary policy in Norway has followed some kind of Taylor rule from 1993, we choose a data sample starting in 1994m1¹². Figure 4.1 shows that some variables display a possible nonstationary behaviour.



Figur 4.1 Time series plots of all variables, 1994m1-2005m7

¹² The data sample starts 1994m1, however, this will in the case of the twelve month inflation rates also include data from 1993.

Testing for unit roots using the augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) tests gives somewhat contradictory results. Both tests reject the hypothesis that the domestic monthly inflation rate is I(1). However, only one test rejects the unit root hypothesis in the imported monthly inflation rate and the unemployment rate. The other variables cannot reject the hypothesis of I(1) in any of the tests. These results are however sensitive to the sample period. Hence, keeping in mind the low power of the unit root tests, and that seasonally adjusted data reduce the power of these tests further, we model the data in levels.

Table 4.1 Augmented Dickey-Fuller test, 1994m1-2005m7^a

	π_d^y t-adj	π_{imp}^y t-adj	π_d^m t-adj	π_{imp}^m t-adj	u t-adj	i t-adj	i* t-adj	e t-adj
Levels	-1,78 (0)	-1,96 (0)	-11,21 *** (0)	-1,86 (11)	-2,58* (0)	-1,46 (1)	-0,59 (1)	-2,36 (1)
1 st difference	-12,20*** (0)	-11,10*** (0)	-10,68*** (10)	8,58*** (10)	-2,86* (4)	-6,62*** (0)	-7,24*** (0)	-9,16*** (0)

^a Except for the level of the oil price and the exchange rate which are tested for nonstationarity with a constant and a trend, the series are tested with a constant only. Critical values are taken from MacKinnon (1996). In parenthesis, the number of lags in the ADF tests. * Significant at the 10% level. ** Significant at the 5% level. *** Significant at the 1% level.

Table 4.2 Phillips Perron test, 1994m1-2005m7^a

	π_d^y t-adj	π_{imp}^y t-adj	π_d^m t-adj	π_{imp}^m t-adj	u t-adj	i t-adj	i* t-adj	e t-adj
Levels	-1,78 (3)	-2,04 (10)	-11,27*** (4)	-12,00*** (21)	-2,29 (8)	-1,16 (6)	-0,80 (8)	-2,00 (1)
1 st difference	-12,19*** (3)	-11,12*** (14)	-68,27*** (36)	-35,96*** (20)	-12,64*** (8)	-6,62*** (2)	-7,85*** (7)	-8,99*** (7)

^a Except for the level of the oil price and the exchange rate which are tested for nonstationarity with a constant and a trend, the series are tested with a constant only. Critical values are taken from MacKinnon (1996). In parenthesis, the Newey-West bandwidth using the Bartlett kernel function. * Significant at the 10% level. ** Significant at the 5% level. *** Significant at the 1% level.

4.1 VAR

While the choice of variables to include in the VAR model was done on the basis of predictive accuracy of the variable in question, the number of lags in the VAR models was

determined using information criteria, the F-form of likelihood ratio test for model reduction. For some models, the lag reduction tests diverged as to which lag order to choose. Then, to minimize the risk of overfitting, we chose the lowest lag order model with more lags than what was suggested by the F-test that did not reject the null of no autocorrelation. Results from tests of residual autocorrelation of the chosen VAR models are reported in Table 4.3, 4.4 and 4.5.

We make static forecasts with the different VAR models at the different horizons for the variables of interest, starting with a data sample from 1994m1 to 2001m3, extending the data sample one observation at a time. From the series of forecasts, we compute the different measures of forecasting accuracy of the different variables at all horizons.

A six-variable VAR model, including the unemployment rate, the domestic and the imported monthly inflation rates, the domestic and the foreign interest rates, and the exchange rate, was chosen to forecast the unemployment rate. When choosing the number of lags to include in the model, the SIC and HQ information criteria indicated choosing only one lag, while the AIC criterion indicated choosing 12 lags. However, the F-test suggested that choosing 8 lags was significantly better than any lower lag model, but at eight lags, the hypothesis of no autocorrelation is rejected. We therefore end up choosing 9 lags, where we accept the hypothesis of no heteroskedasticity and no autocorrelation. The hypothesis of normality however, is rejected at all lags. This is mainly due to excess kurtosis in the interest rate equations.

At all horizons, the same five-variable VAR model containing domestic and foreign interest rates, the unemployment rate and the exchange rate, as well as the domestic monthly inflation rate, was chosen as the best model to forecast the monthly domestic inflation rate. Again, the AIC criterion suggested using 12 lags, while the SIC and HQ suggested using 1. The F-test found that four lags were significantly better than choosing fewer lags. However, at four lags, the autocorrelation in the model is significant. We therefore chose the model at the different horizons with more than four lags, which minimizes the forecasting errors and where the hypothesis of no autocorrelation was not rejected. Again some non-normality remained in the two interest rate equations, and was again due to excess kurtosis.

When forecasting the twelve month domestic inflation rate at three, six and 12 months horizons, the same five-variable VAR was chosen as the one forecasting the monthly inflation rate, except for the obvious replacement of the monthly rate of inflation with the twelve month rate. Again, the tests differed as to how many lags to choose. AIC suggested 12, SIC 1, and HQ and the F-test suggested 2 lags. But at two lags the residuals are correlated, while 3 lags ensure the lowest forecasting error, and we accept our null hypothesis of no autocorrelation. At the six months horizon, we included the yearly rate of imported inflation in the model, as this improved the forecasting accuracy. The tests of lag order again diverge, and for the same reasons as above, we chose three lags. There were some remaining non-normality also in these two models at three lags, but again this was due to excess kurtosis.

We also tested the forecasting accuracy of the different variables when including the price of Brent blend in the VAR models. However this did not improve the prediction errors, it in fact leads us to reject the hypothesis of no autocorrelation at all lags. A linear time trend was found to be insignificant and therefore left out of the VAR models.

Table 4.3 Tests of residual autocorrelation in VAR, u

	3 month horizon	6 month horizon	12 month horizon
Model	$\pi_d^m, \pi_{imp}^m, i, i^*, u, e$	$\pi_d^m, \pi_{imp}^m, i, i^*, u, e$	$\pi_d^m, \pi_{imp}^m, i, i^*, u, e$
Sample	1994m1-2005m4	1994m1-2005m1	1994m1-2004m7
Test statistic	38,05	36,13	33,84
p-value	0,3764	0,4627	0,5719

Table 4.4 Tests of residual autocorrelation in VAR, Π_d m/m

	3 month horizon	6 month horizon	12 month horizon
Model	π_d^m, i, i^*, u, e	π_d^m, i, i^*, u, e	π_d^m, i, i^*, u, e
Sample	1994m1-2005m4	1994m1-2005m1	1994m1-2004m7
Test statistic	18,19	17,71	14,83
p-value	0,8341	0,8543	0,9452

Table 4.5 Tests of residual autocorrelation in VAR, Π_d y/y

	3 month horizon	6 month horizon	12 month horizon
Model	π_d^y, i, i^*, u, e	$\pi_d^y, \pi_{imp}^y, i, i^*, u, e$	π_d^y, i, i^*, u, e
Sample	1994m1-2005m4	1994m1-2005m1	1994m1-2004m7
Test statistic	24,31	36,41	24,34
p-value	0,5015	0,4472	0,4997

4.2 ARMA

The Box-Jenkins approach or some objective measure of model suitability is usually applied to find the right ARMA form to model the stationary series, before a more formal assessment of only a few models is undertaken. However, to avoid the subjective nature of the Box-Jenkins approach, and because we want to evaluate the different models at different time horizons which make the comparison of e.g. different information criteria difficult, we choose to evaluate the models on the basis of predictive ability. A rule of thumb when choosing the maximum order of AR and MA lags in the ARMA models is to select the seasonal span minus one. However, assuming invertibility of the MA process, we test for all combinations of 11 autoregressive lags and 3 moving average lags, all in all a total of 46 models for all variables at all horizons. Because we use seasonally adjusted time series, we do not model the seasonal pattern. As some series still contain some seasonal variations, this approach could have been useful, but was left out to be consistent, and limit the number of tested models.

With a data sample from 1994m1 to 2001m3, we make forecasts of the different variables over the different horizons. By extending the sample by one observation at a time, making forecasts, and iterating forwards, we obtain a series of forecasts for each variable and each ARMA model at the different horizons. Models which rejected the hypothesis of no autocorrelation in more than three sample periods, or which violated the stability condition¹³ were rejected. Results from the tests of residual autocorrelation in the chosen ARMA models are reported in Table 4.6, 4.7 and 4.8.

¹³ All roots of the characteristic polynomial must lie outside the unit circle.

Table 4.6 Tests of residual autocorrelation in ARMA, u

	3 month horizon	6 month horizon	12 month horizon
Model	(7 0 2)	(9 0 1)	(9 0 1)
Sample	1994m1-2005m4	1994m1-2005m1	1994m1-2004m7
p-value	0,7401	0,1230	0,1969

Table 4.7 Tests of residual autocorrelation in ARMA, Π_d m/m

	3 month horizon	6 month horizon	12 month horizon
Model	(7 0 1)	(7 0 1)	(2 0 3)
Sample	1994m1-2005m4	1994m1-2005m1	1994m1-2004m7
p-value	0,0803	0,0741	0,0888

Table 4.8 Tests of residual autocorrelation in ARMA, Π_d y/y

	3 month horizon	6 month horizon	12 month horizon
Model	(4 0 1)	(4 0 1)	(5 0 2)
Sample	1994m1-2005m4	1994m1-2005m1	1994m1-2004m7
p-value	0,0824	0,0433	0,1256

4.3 Random Walk

We perform forecasts of random walks by estimating the ARIMA model (0,1,0) on u , Π_i m/m and Π_i y/y. As with the VAR and ARMA models, we start with a data sample from 1994m1 to 2001m3, produce forecasts, then expanding the sample observation by observation until we have a series of forecasts of the different horizons. The forecast accuracy of the random walk models at the different horizons are used as a benchmark to the forecasts produced by the VAR and ARMA models.

5. Forecasting performance

Evaluating the accuracy of forecasts is usually done by analysing the difference between the forecasts and the observed series, assuming a quadratic loss function, and thus assuming the loss function to be symmetric. However, assessing the economic loss associated with the different forecast by the usual statistical measures may in some cases be questionable. Depending on the intended use of the forecasts, realistic loss functions may for example depend on direction of change, market and country timing or some utility function. While some test allows for different loss functions, e.g. the test proposed by Diebold and Mariano (1995) which tests the significance of the difference of two competing forecasts, Granger and Newbold (1986) points out that the least-squares approach, corresponding to a quadratic loss function will in most practical cases be defensible, especially if the primary interest is in comparing two forecasts of the same variable. We will evaluate the forecasting accuracy of the different models by a relative approach, comparing the root mean square forecast error of the different models, thus assuming a quadratic loss function.

5.1 Measures of forecasting performance

From the series of forecast of the different variables at all horizons, we calculate the following measures of forecasting accuracy:

Mean Square Forecasting Error (MSFE)

Let f_t be the h in-sample forecasts of the observed y_t . The mean square forecasting error is then:

$$(5.1) \quad MSFE = \frac{1}{h} \sum_t (y_t - f_t)^2$$

Root Mean Square Forecasting Error (RMSFE)

This is simply the root of the mean square forecasting error:

$$(5.2) \quad RMSFE = \left(\frac{1}{h} \sum_t (y_t - f_t)^2 \right)^{\frac{1}{2}}$$

This measure of the forecasting accuracy is widely used, and considered appropriate when the cost of errors increases with the square of the error. However; these measures of predictive accuracy are sensitive to the occasional large error, that is, the squaring weights large forecasting errors more than it does small.

Mean Absolute Percentage Forecasting Error (MAPFE)

The mean absolute forecasting is calculated as:

$$(5.3) \quad MAPFE = \frac{100}{h} \sum_t \left| \frac{y_t - f_t}{y_t} \right|$$

The MAPFE is an appropriate measure of predictive accuracy when the cost of errors is more related to the percentage error than to the numerical size of the error. This measure of predictive accuracy has the advantage of being dimensionless; however it is worth noting that for any process with first moment close to zero, MAPFE can approach infinity.

As the variables forecasted are rates, we focus on the RMSFE, as this is measured in the same units as the data, and thus is representative of the size of the “typical” error.

5.2 Evaluating forecasting performance

The results of the forecasting of the different variables are shown in Table 5.1, 5.2 and 5.3 below. At all horizons, forecasts of the unemployment rate from the VAR model and the ARMA models outperform a random walk. However, the univariate models perform relatively similar at all horizons, and all models perform relatively similar at the very short horizon. The forecast error of the VAR model increases less than the forecast errors for the ARMA and RW models when the horizon increases. The finding that a multivariate model outperforms the univariate models is supported by the findings in the analysis of the transmission mechanism of monetary policy. Thus, imposing some economic relationship allowing for the interdependence we expect to find among economic variables, and including more information enhances predictive accuracy of the unemployment rate.

Table 5.1 Forecasting u

	VAR ^a	ARMA	RW
3-step ahead	$\pi_d^m, \pi_{imp}^m, i, i^*, u, e$	(7 0 2)	(0 1 0)
MSFE	0,02	0,03	0,03
RMSFE	0,15	0,17	0,16
MAPFE	3,18	4,00	3,83
6-step ahead	$\pi_d^m, \pi_{imp}^m, i, i^*, u, e$	(9 0 1)	(0 1 0)
MSFE	0,06	0,08	0,08
RMSFE	0,25	0,29	0,28
MAPFE	6,20	6,81	6,82
12-step ahead	$\pi_d^m, \pi_{imp}^m, i, i^*, u, e$	(9 0 1)	(0 1 0)
MSFE	0,16	0,26	0,27
RMSFE	0,40	0,51	0,52
MAPFE	9,93	11,69	12,18

^a The VAR models were estimated using 9 lags at all horizons.

Table 5.2 Forecasting Π_d m/m

	VAR ^a	ARMA	RW
3-step ahead	π_d^m, i, i^*, u, e	(7 0 1)	(0 1 0)
MSFE	0,04	0,02	0,03
RMSFE	0,20	0,14	0,18
MAPFE	117,32	128,29	120,31
6-step ahead	π_d^m, i, i^*, u, e	(7 0 1)	(0 1 0)
MSFE	0,04	0,02	0,03
RMSFE	0,20	0,14	0,17
MAPFE	187,88	173,25	179,00
12-step ahead	π_d^m, i, i^*, u, e	(2 0 3)	(0 1 0)
MSFE	0,04	0,02	0,05
RMSFE	0,19	0,15	0,22
MAPFE	172,37	212,30	247,35

^a At the 12-months horizon we used 8 lags in the VAR model, while at both the 6- and 3-months horizons we estimate the VAR models using 5 lags.

When it comes to the inflation rates, the results are somewhat different. The ARMA models of the monthly inflation rate have the lowest RMSFE at all horizons. However, the minimum MAPFE differ between models. The high percentage errors are no surprise, as the mean of monthly inflation under the sample period is 0.2 percent. A noteworthy result is the equality of the RMSFE for the ARMA models at all horizons. Even though the MAPFE increases with an increasing horizon, the RMSFE does not.

As with the monthly inflation rate, the ARMA models predict the twelve month inflation rate better than the VAR and the RW at all horizons. From the effects of a contractionary monetary policy shock on inflation in Section 2.2, we found inflation to be persistent, with the full effect of the shock after two and a half years. Thus, there is no surprise in the predictive performance of the ARMA models. In particular, the VAR performs poorly at the 3 month horizon, probably due to the dynamics in the other variables in the model. That even the random walk performs relatively similar to the ARMA in predicting the inflation rate supports the assumption of nominal rigidities.

Table 5.3 Forecasting $\Pi_d y/y$

	VAR ^a	ARMA	RW
3-step ahead	π_d^y, i, i^*, u, e	(4 0 1)	(0 1 0)
MSFE	1,07	0,16	0,20
RMSFE	1,03	0,40	0,45
MAPFE	35,75	14,60	17,19
6-step ahead	$\pi_d^y, \pi_{imp}^y, i, i^*, u, e$	(4 0 1)	(0 1 0)
MSFE	0,39	0,38	0,50
RMSFE	0,62	0,62	0,71
MAPFE	28,58	27,25	32,13
12-step ahead	π_d^y, i, i^*, u, e	(5 0 2)	(0 1 0)
MSFE	0,75	0,37	1,37
RMSFE	0,87	0,80	1,17
MAPFE	45,96	41,10	60,51

^a The VAR models were estimated using 3 lags at all horizons.

While we have chosen to predict the different variables at the different horizons by maximizing the predictive accuracy, thereby allowing for different VAR and ARMA models at the different horizons, this can be time-consuming in applied use. The difference in predictive accuracy between the different models estimated is relatively similar. Hence, choosing a different model, or using a single model for all horizons will not change the above results significantly.

Another reason for ARMA models to be reasonable only in the short run is that, as shown in Section 3.2, the forecasting errors of an ARMA model are correlated. These models will therefore have a tendency to either over- or underpredict the observed values of the time series, which will increase the forecast errors and decrease the predictive power at an increasing horizon.

When forecasting with ARMA and VAR models, parameter instability in the data sample may cause poor out-of-sample forecasts. While all models are found to be stationary, tests of parameter stability did indicate that some VAR models had time varying coefficients. But as this was mainly in the first year of the sample, this was considered less important. The VAR models are not re-specified at any point of the estimation period, hence our recursive estimation scheme imply model stability over the entire data sample. This assumption does probably not hold.

6. Concluding remarks

The success of macroeconomic models in a central bank perspective depends on how useful they are in helping policymakers to conduct monetary policy. A thorough understanding of the transmission mechanism of monetary policy is crucial in order for the central bank to be able to undertake the appropriate policy responses to shocks to the economy. Models of the transmission mechanism that identify the effects of a monetary policy shock are therefore important to policy makers. For an economy operating an inflation targeting regime, policymakers need to be forward-looking and make projections of the economic development. Models for short term analysis will in such a framework contribute to a more profound knowledge of the current situation of the economy, and to what forces and disturbances that drive the current economic development.

To investigate the predictive accuracy of some commonly used short term forecasting models, we make forecasts of the domestic consumer price inflation and the unemployment rate using unrestricted VAR models, ARMA models and random walk. We use a short run forecasting horizon of three to twelve months. While univariate models highlight only the statistical properties of a particular data series, multivariate models take into account more information and implicitly impose an economic relationship when choosing the variables in the model. The multivariate VAR models are best at predicting the unemployment rate at all horizons, outperforming the random walk, but also the ARMA models. As for the inflation rates, ARMA outperforms the random walk at all horizons, while the VAR models perform similar in predicting the twelve month inflation rate at the six and twelve month horizon. The reason for the forecasting performance of the inflation rate of the ARMA models may be attributed to the persistence in inflation.

By recursive identification of the VAR model, we analyze the effect of a shock to monetary policy in an open economy. The recursive identification imposes a standard structure on the variables in the VAR, in that the macroeconomic variables react to shocks to monetary policy with a lag, while we allow for a contemporaneous effect of a shock to the macroeconomic variables on monetary policy. Our results show that a shock to monetary policy increases the domestic interest rate temporarily, while the exchange rate appreciates

immediately, before it slowly depreciates back to baseline. Unemployment increases for a period of five quarters, while the effect on inflation is sluggish, but negative, with monetary policy having its full effect after two and a half years.

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Appendicies

Appendix A: Data and sources

(i) P_d - Domestic consumer price index

The seasonally adjusted domestic consumer price index is adjusted for changes in energy prices and taxes (CPI-ATED). Source: Statistics Norway and Norges Bank.

(ii) P_{imp} - Imported consumer price index

The seasonally adjusted imported consumer price index is adjusted for changes in energy prices and taxes (CPI-ATEIMP). Source: Statistics Norway and Norges Bank.

(iii) u - Unemployment

Registered unemployment rate. Seasonally adjusted. Source: Aetat, the Norwegian Public Employment Service.

(iv) i - Domestic interest rate

The three month domestic effective nominal interest rate (NIBOR). Source: Reuters.

(v) i^* - Foreign interest rate

The three month foreign effective nominal interest rate calculated as a trade weighted sum of the interest rates of Norway's four largest trading partners (Euro-zone, Sweden, United Kingdom and USA). Source: Reuters and OECD.

(vi) e - Exchange rate

The real exchange rate. The nominal exchange rate against Norway's 25 most important trading partners, adjusted by the ratio between the consumer price index of the 23 most important trading partners and the domestic consumer price index (CPI-ATE). Source: Norges Bank and Statistics Norway.

Appendix B: Stationarity, unit roots and cointegration

A stochastic process $\{X_t\}$ is strictly stationary if the joint distribution of any set of n observations exists, and is independent of time. In applied work however, weak stationarity is usually sufficient, which requires only the first and second order moments of the process to exist and be time invariant. We have:

$$(B.1) \quad E[X_t] = \mu \quad \forall t$$

$$(B.2) \quad E(X_t, X_{t-j}) = \begin{cases} \sigma^2 & \text{for } j = 0 \\ 0 & \text{for } j \neq 0 \end{cases}$$

We see that the covariance between X_t and X_{t-j} (or X_{t+j}) depends on j only, and not t . Hence, the effect of any given shock will eventually die out.

An MA(q) process can be expressed, using the lag operator, as

$$(B.3) \quad \phi(L)X_t = \theta(L)\varepsilon_t$$

where $\{\varepsilon_t\}$ is a white noise process. As these errors are uncorrelated across time, we get the first and second order moments as

$$(B.4) \quad E[\varepsilon_t] = 0 \quad \forall t,$$

which implies

$$(B.5) \quad E[X_t] = 0 \quad \forall t$$

$$(B.6) \quad \text{var}(X_t) = \sigma^2 \sum_{i=0}^p \beta_i^2,$$

where β_0 is equal to unity.

The covariance can be shown to be:

$$(B.7) \quad \text{cov}(X_t, X_{t-k}) = \begin{cases} \sigma^2 \sum_{i=0}^{p-k} \beta_i \beta_{i+k} & \vee \quad k = 0, 1, \dots, p \\ 0 & \vee \quad k > p \end{cases}$$

Hence, the MA(q) process is always weakly stationary.

Whether an ARMA process is stationary therefore depends only on the autoregressive part of the model. An AR(p) process can be expressed as:

$$(B.8) \quad X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \dots + \alpha_p X_{t-p} + \varepsilon_t,$$

which by use of the lag operator can be rewritten to yield:

$$(B.9) \quad \phi(L)X_t = \varepsilon_t$$

The corresponding characteristic equation is

$$(B.10) \quad \phi(L) = 1 - \alpha_1 L - \alpha_2 L^2 - \dots - \alpha_p L^p$$

If the roots of the characteristic equation lie outside the unit circle, the ARMA(p,q) process is stationary¹⁴.

The corresponding characteristic equation of a VAR model can be shown to be

$$(B.11) \quad \left| \mathbf{I}_n - \Phi_1 z - \Phi_2 z^2 - \dots - \Phi_p z^p \right| = 0$$

Hence, a VAR model is stationary if all characteristic roots, or eigenvalues, of the determinantal equations lie outside the unit circle.

¹⁴ The roots of the characteristic equation may be complex. If so, they are of the form $a \pm bi$ where $i = \sqrt{-1}$. The unit circle refers to the two-dimensional set of values of a and b defined by $a^2 + b^2 = 1$, which defines a circle centred at the origin with a radius of one.

B.1 Dickey-Fuller and Augmented Dickey-Fuller tests

In the Dickey-Fuller (DF) models, X_t is assumed to follow an AR(1) process. But if X_t follows an AR(p) process ($p > 1$), the error term will be autocorrelated. However the Dickey-Fuller distributions are based on the error terms being white noise. This is taken into account in the Augmented Dickey-Fuller (ADF) tests. The ADF-tests with a constant and with a constant and a trend is specified as follows:

$$(B.12) \quad \Delta X_t = \mu_a + (\rho_a - 1)X_{t-1} + \sum_{k=1}^{K-1} \gamma_k \Delta X_{t-k} + u_t \quad u_t \sim IID(0, \sigma^2)$$

$$(B.13) \quad \Delta X_t = \mu_b + (\rho_b - 1)X_{t-1} + \lambda_b t + \sum_{k=1}^{K-1} \gamma_k \Delta X_{t-k} + \varepsilon_t \quad \varepsilon_t \sim IID(0, \sigma^2)$$

Critical values are taken from MacKinnon (1996). If we choose too few lags, we run the risk of rejecting a true null hypothesis, but on the other hand if we choose too many, we reduce the strength of the tests. If the data contains seasonal variations, this has to be taken into account. Ghysels, Lee and Noh (1994) recommend including as many lags as necessary to take the seasonalities into account, even if the data is seasonally adjusted, as such data still may exhibit seasonal patterns. Using monthly data, we include 12 lags in the ADF tests. The choice of lag length is done by minimizing the Schwarz Information Criteria over all 13 models.

B.2 Phillips-Perron test

The Phillips-Perron (PP) test estimates the standard DF test, and adjusts the t-ratio of the α -coefficient so that serial correlation does not affect the asymptotic distribution of the test statistic. The PP-test is based on the test-statistic:

$$(B.14) \quad \tau_\alpha = t_\alpha \left(\frac{\gamma_0}{f_0} \right)^{1/2} - \frac{T(f_0 - \gamma_0)(se(\hat{\alpha}))}{2f_0^{1/2}\sigma}$$

, where $\hat{\alpha}$ is the coefficient estimate of the lagged dependent variable, T is sample size, $se(\hat{\alpha})$ is the estimated standard error of the coefficient, σ is the regression standard error, γ_0 is the error variance of the DF-regression, and f_0 is an estimate of the residual spectrum at frequency zero.

When performing an PP test, we must choose an estimation method for f_0 . We will be using the Bartlett kernel function using a Newey-West bandwidth parameter. The asymptotic distribution of the PP based t-statistic coincides with that of the ADF statistic. Critical values are taken from MacKinnon (1996).

B.3 Cointegration

A $(n \times 1)$ vector \mathbf{y}_t is said to be cointegrated if each of its elements individually are $I(1)$ and if there exists a nonzero $(n \times 1)$ vector such that $\beta' \mathbf{y}_t$ is stationary. When this is the case, β is called a *cointegrating* vector. Consider a VAR of order p :

$$(B.15) \quad \mathbf{y}_t = A_1 \mathbf{y}_{t-1} + \dots + A_p \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t,$$

where \mathbf{y}_t is a k -vector of nonstationary $I(1)$ variables, and $\boldsymbol{\varepsilon}_t$ is a vector of serially uncorrelated residuals with covariance matrix $\boldsymbol{\Omega}$. This can be rewritten as

$$(B.16) \quad \Delta \mathbf{y}_t = \Pi \mathbf{y}_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta \mathbf{y}_{t-i} + \boldsymbol{\varepsilon}_t,$$

where

$$(B.17) \quad \Pi = \sum_{i=1}^p A_i - I$$

$$(B.18) \quad \Gamma_i = - \sum_{j=i+1}^p A_j$$

If the coefficient matrix Π has reduced rank, that is $r < k$, then there exists $k \times r$ matrices α and β , such that $\Pi = \alpha \beta'$ and $\beta' \mathbf{y}_t$ is $I(0)$. r is the number of cointegrating relations and each column of β is the cointegration vector. To test for cointegration, using the Johansen test, the coefficient matrix Π is estimated from an unrestricted VAR model, and then test whether one can reject the restrictions implied by the reduced rank of Π . For a detailed exposition of the Johansen test of cointegration, see Hamilton (1994, chapter 20.2).

